Overview of the Student's T-Test

Introduction

The student's t-test is a test of statistical significance between the means of two groups of data (a two sample t-test) or between the mean one group of data vs an accepted value (a one sample t-test) to determine if any noticeable differences in the means are statistically significant or do to simply random error. This idea of hypothesis testing was originally developed by statistician William Sealy Gosset and published in the English research journal *Biometrika* around 1908 under the pseudonym *Student*. The two main assumptions underlying the student's t-test is that the data sets have similar variances (s^2) and that these data sets have a normal distribution (a bell curve).

How to Perform a T-Test

1. Identify your hypothesis (H) and your null hypothesis (H_0). Most of the time the hypothesis is something along the lines of "The mean of Data Set A is significantly different than the mean of Data Set B" while the null hypothesis is the inverse or something like "There is no significant difference between the means of Data Set A and Data Set B".
2. Calculate the means and standard deviations of each data set to be used in the t-test.
3. Identify the type of t-test to be performed. These include:
    a. Paired vs Unpaired t-test: A paired t-test is used to compare related data sets such as a pre-test vs post-test or cholesterol before and after administration of a drug. In contrast an unpaired t-test has unrelated data sets.
    b. One sample vs two sample: As described above in the introduction
    c. Two tailed vs One tailed: A two-tailed t-test just cares about if Data Set A is significantly different than Data Set B (or the accepted value) while a one-tailed t-test cares about the mean of Data Set A being either significantly greater than or significantly less than (need to pick one) the mean of Data Set B (or the accepted value)
4. Apply the correct formula based on the type of t-test performed to calculate a t-statistic (t_{stat}). Some relevant equations and their symbol meanings are shown below

$$t_{stat} = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$$

Equation 1: A One Sample T-Test

$$\bar{x} = mean\ of\ data\ set$$

$$\mu = accepted\ value$$

$$s = standard\ deviation\ of\ data\ set$$

$$n = number\ of\ samples\ in\ data\ set$$

$$t_{stat} = \frac{\overline{x_D}}{\frac{s_D}{\sqrt{n}}}$$

Equation 2: A Paired T-Test

$$\overline{x_D} = mean\ of\ the\ differences\ between\ each\ paired\ sample$$

$$s_D = standard\ deviation\ of\ the\ differences\ between\ each\ paired\ sample$$

$$n = number\ of\ paired\ samples\ in\ data\ set$$

$$t_{stat} = \frac{\overline{x_A} - \overline{x_B}}{\sqrt{\frac{s_A^2}{n_A} + \frac{s_B^2}{n_b}}}$$

Equation 3: An Independent (Unpaired Two Sample) T-Test

$$\overline{x_A} = mean\ of\ data\ set\ A$$

$$\overline{x_B} = mean\ of\ data\ set\ B$$

$$s_A = standard\ deviation\ of\ data\ set\ A$$

$$s_B = standard\ deviation\ of\ data\ set\ B$$

$$n_A = number\ of\ samples\ in\ data\ set\ A$$

$$n_B = number\ of\ samples\ in\ data\ set\ B$$

5. Determine your $t_{critical}$ using the t-test chart (next page)
   a. Going down the left side, df stands for “degrees of freedom” and corresponds to:
      i. $n - 1$ for a one sample t-test
      ii. $n_D - 1$ for a paired sample t-test
      iii. $n_A + n_B - 1$ for an independent t-test
   b. Across the top is your significance level (α). We use 0.05 or smaller. Make sure to choose the respective tailed option
   c. Where the two meet is your $t_{critical}$.

Table T Critical Values of the *t* Distribution

df	One-Tail = .4 Two-Tail = .8	.25 .5	.1 .2	.05 .1	.025 .05	.01 .02	.005 .01	.0025 .005	.001 .002	.0005 .001
1	0.325	1.000	3.078	6.314	12.706	31.821	63.657	127.32	318.31	636.62
2	0.289	0.816	1.886	2.920	4.303	6.965	9.925	14.089	22.327	31.598
3	0.277	0.765	1.638	2.353	3.182	4.541	5.841	7.453	10.214	12.924
4	0.271	0.741	1.533	2.132	2.776	3.747	4.604	5.598	7.173	8.610
5	0.267	0.727	1.476	2.015	2.571	3.365	4.032	4.773	5.893	6.869
6	0.265	0.718	1.440	1.943	2.447	3.143	3.707	4.317	5.208	5.959
7	0.263	0.711	1.415	1.895	2.365	2.998	3.499	4.029	4.785	5.408
8	0.262	0.706	1.397	1.860	2.306	2.896	3.355	3.833	4.501	5.041
9	0.261	0.703	1.383	1.833	2.262	2.821	3.250	3.690	4.297	4.781
10	0.260	0.700	1.372	1.812	2.228	2.764	3.169	3.581	4.144	4.587
11	0.260	0.697	1.363	1.796	2.201	2.718	3.106	3.497	4.025	4.437
12	0.259	0.695	1.356	1.782	2.179	2.681	3.055	3.428	3.930	4.318
13	0.259	0.694	1.350	1.771	2.160	2.650	3.012	3.372	3.852	4.221
14	0.258	0.692	1.345	1.761	2.145	2.624	2.977	3.326	3.787	4.140
15	0.258	0.691	1.341	1.753	2.131	2.602	2.947	3.286	3.733	4.073
16	0.258	0.690	1.337	1.746	2.120	2.583	2.921	3.252	3.686	4.015
17	0.257	0.689	1.333	1.740	2.110	2.567	2.898	3.222	3.646	3.965
18	0.257	0.688	1.330	1.734	2.101	2.552	2.878	3.197	3.610	3.922
19	0.257	0.688	1.328	1.729	2.093	2.539	2.861	3.174	3.579	3.883
20	0.257	0.687	1.325	1.725	2.086	2.528	2.845	3.153	3.552	3.850
21	0.257	0.686	1.323	1.721	2.080	2.518	2.831	3.135	3.527	3.819
22	0.256	0.686	1.321	1.717	2.074	2.508	2.819	3.119	3.505	3.792
23	0.256	0.685	1.319	1.714	2.069	2.500	2.807	3.104	3.485	3.767
24	0.256	0.685	1.318	1.711	2.064	2.492	2.797	3.091	3.467	3.745
25	0.256	0.684	1.316	1.708	2.060	2.485	2.787	3.078	3.450	3.725
26	0.256	0.684	1.315	1.706	2.056	2.479	2.779	3.067	3.435	3.707
27	0.256	0.684	1.314	1.703	2.052	2.473	2.771	3.057	3.421	3.690
28	0.256	0.683	1.313	1.701	2.048	2.467	2.763	3.047	3.408	3.674
29	0.256	0.683	1.311	1.699	2.045	2.462	2.756	3.038	3.396	3.659
30	0.256	0.683	1.310	1.697	2.042	2.457	2.750	3.030	3.385	3.646
40	0.255	0.681	1.303	1.684	2.021	2.423	2.704	2.971	3.307	3.551
60	0.254	0.679	1.296	1.671	2.000	2.390	2.660	2.915	3.232	3.460
120	0.254	0.677	1.289	1.658	1.980	2.358	2.617	2.860	3.160	3.373
∞	0.253	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

Source: From *Biometrika Tables for Statisticians,* Vol. 1, Third Edition, edited by E. S. Pearson and H. O. Hartley, 1966, p. 146. Reprinted by permission of the Biometrika Trustees.

6. Interpret your results
    a. If $t_{stat} > t_{critical}$ then there is a significant difference (we reject the null hypothesis). If our significance level was 0.05 what that means is that we are 95% sure that there is a real difference and only a 5% chance that the difference is due to random error. This is why the smaller the significance level the better!
    b. If $t_{critical} > t_{stat}$ then there is not a significant difference (we fail to reject the null hypothesis…..notice we don't accept the actual hypothesis)